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Rare Electroweak Processes $K_L^0 \to \mu^+\mu^-$ and $K_L^0 \to \gamma\gamma$ and Heavy Top Quark

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Abstract

A brief overview of the recent measurements of the branching ratio of the rare $K_L^0 \to \mu^+\mu^-$ decay in the context of their agreement with the Standard Model (SM) is given. It is shown that KEK result well correlates with the SM and B-physics, whereas the BNL results are in conflict with the SM with the heavy top quark.

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For a long time the rare electroweak decays $K_L^0 \to \mu^+\mu^-$ and $K_L^0 \to \gamma\gamma$ come to the attention of physicists. The connection between absorptive width of $K_L^0 \to \mu^+\mu^-$ decay calculated from imagine part of an amplitude and total $K_L^0 \to \gamma\gamma$ decay width follow from the known unitarity relation:

$$Br_{abs}(K_L^0 \to \mu^+\mu^-) \simeq 1.2 \cdot 10^{-5}\, Br(K_L^0 \to \gamma\gamma) = (6.8 \pm 0.3) \cdot 10^{-9}. \quad (1)$$

Here we used the experimental value of $Br(K_L^0 \to \gamma\gamma) = (5.70 \pm 0.27) \cdot 10^{-4}$. This minimal value allowed by theory is known as the unitarity limit. It is curiously to emphasize that the amplitude of $K_L^0 \to \mu^+\mu^-$ decay consists of two comparable contributions. The first one is due to long distances ($r \sim 1/m_K$), where the light quark contribution is essential, and the other come from short distances ($r \leq 1/m_W$), where the contributions of heavy quarks dominate. At the first time this fact was shown in the paper of M. Voloshin and E. Shabalin [1], where the estimation of the c-quark mass from the width of the mentioned above decay in the framework of the SM with two generations was obtained. As time passed, it became aware the importance of $K_L^0 \to \mu^+\mu^-$ decay as one more source for the top quark mass estimation (see, for example, Ref. [2, 3]). In our papers [4, 5, 6] the total amplitude of this process has been calculated within the quark model approach.

In the quark approach $K_L^0 \to \mu^+\mu^-$ amplitude is a sum of one-loop ($1L$) and two-loop ($2L$) contributions. The first one (through W and Z) due to short distances $\sim 1/m_W$ where top quark contribution dominates. As for the $2L$ contribution with two photon intermediate state, medium ($\sim 1/m_c$) and rather long ($\geq 1/m_K$) distances are essential. The evidence of the top quark production at the $p\bar{p}$ collider (CDF Collab., FNAL) [7] and also a further precision of the estimations of V_{ub} and V_{cb} CKM matrix elements improved by ARGUS and CLEO [8] allow us to overview the modern status of $K_L^0 \to \mu^+\mu^-$ decay. Namely, calculating the total decay amplitude in the framework of the SM one can obtain the restriction on the decay width and, by this means, investigate the agreement of the recent experimental data with the SM.

We pointed out the principal importance of the relative sign between $1L$ and $2L$ contributions. Let us note that in the terms of the bare quarks the total decay amplitude contains these contributions with opposite signs [1]. The obtained by us amplitude [4] in the limit $m_u^2 \ll m_K^2/4$, $m_c^2 \gg m_K^2/4$ for the current u- and c-quarks agrees with the result of Voloshin and Shabalin [1]. However we emphasize that it is necessary to account the QCD

corrections to the effective four-quark vertex in order to obtain a realistic result for $2L$ contribution. To that end, we used the renormgroup method by Vainstein, Zakharov and Shifman [9] for the mass scale μ down to the typical hadronic scale $\mu_0 \simeq 2\Lambda$ ($\alpha_{st}(\mu_0) = 1$). We have developed also a fenomenological method of the estimation of the QCD corrections on the small scale interval $\mu_0 \leq \mu \leq m_K$ [5]. To test the reliability of our method, we calculated $\Gamma(K_L^0 \to e^+e^-\gamma)/\Gamma(K_L^0 \to \mu^+\mu^-\gamma)$ [6] and showed that our result is closed to one obtained within the phenomenological pole model [3]. Certainly we do not pretend to obtain an integral accuracy better than $30 \div 40\%$ in the description of the contributions of relatively long distances ($r \leq 1/\mu_0$). However, the sign between $1L$ and the real part of $2L$ contributions is fixed sufficiently reliable by this way. Our main result is that the real part of $2L$ contribution changes the sign if the QCD corrections take into account. The change of the sign is connected with the behaviour of the integral over the u-quark loop scale. This integral involves multiplicatively the QCD formfactor of $(V-A)$ four-quark vertex which becomes sufficiently large (more than unit in modulus) and negative number on the interval $2\Lambda \leq \mu \leq m_K$ [9].

The expression for the total $K_L^0 \to \mu^+\mu^-$ amplitude obtained by this way has the form [5]:

$$
\begin{aligned}
\mathcal{M}(K_L^0 \to \mu^+\mu^-) &\simeq -10^{-3}\,\mathcal{N}\,\{+[(5.6 \pm 2.0) - i(44.7 \pm 0.9)] \\
&\quad +2 + 10^3\,\frac{F(m_t^2/m_W^2)}{\sin^2\theta_W}\,\frac{\Re(V_{td}^*V_{ts})}{\Re(V_{cd}^*V_{cs})}\}, \qquad (2) \\
F(x) &= \frac{x}{4}\left[\frac{4-x}{1-x} + \frac{3x\ln x}{(1-x)^2}\right],
\end{aligned}
$$

where $\mathcal{N} = (\alpha/4\pi)\,G_F\,F_K\,m_\mu \sin 2\theta_C(\bar{\mu}\gamma_5\mu)$, $F(x)$ is the well-known function [10], F_K is the formfactor of the K-meson, m_μ is the muon mass, θ_C and θ_W are the Cabibbo and Weinberg angles respectively. The first term in the curly braces describes the $2L$ contribution. We pretend only on the calculation of the real part of $2L$ contribution, and take the imagine part from the unitarity relation (1). The second and third terms of the amplitude (2) describe the c- and t-quark contributions respectively.

It should be noted that our expression (2) for the total decay amplitude is in contradiction with the result by Ko [11] in which the relative sign between the first and others terms of the amplitude is negative, whereas in our expression it is the same. The method developed by Ko [11, 12] has

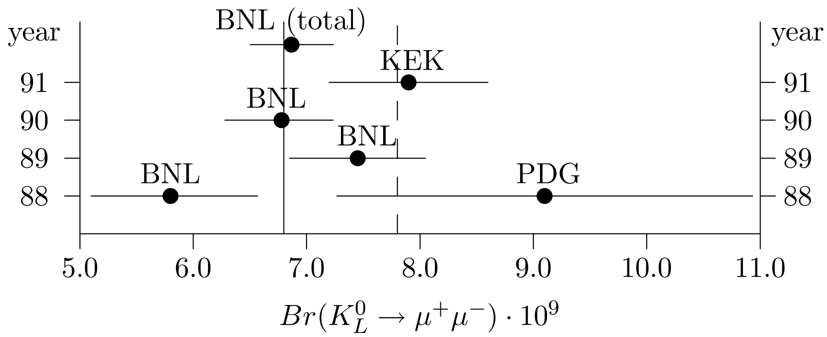

Figure 1: Measurements of the branching ratio of $K_L^0 \to \mu^+\mu^-$ from BNL E791 and KEK E137 data. The solid line is the unitarity limit. The region to the right of the dash line agrees with the SM.

disadvantage. Namely, in these papers the dependence of the meson vertex formfactors (for example, πVV) on the meson loop scale was neglected.

To obtain the restriction on $K_L^0 \to \mu^+\mu^-$ decay width, we used the resent experimental data on the top quark mass [7]

$$m_t = 174 \pm 10^{+13}_{-12} = 174 \pm 16\, GeV$$

and on the parameters of CKM matrix in the Wolfenstain representation [8]

$$\begin{aligned} &\lambda = \sin\theta_C \simeq 0.22, \qquad A = 0.86 \pm 0.10, \\ &\sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09 \qquad \text{which gives} \qquad (1-\rho) \geq 0.64 \pm 0.09. \end{aligned}$$

From Eq. (2) the lower limit on the $K_L^0 \to \mu^+\mu^-$ decay width is following:

$$\begin{aligned} &\Delta Br(K_L^0 \to \mu^+\mu^-) \cdot 10^9 = [Br(K_L^0 \to \mu^+\mu^-) \cdot 10^9 - 6.8] \geq \\ &0.95(1 \pm 0.13 \pm 0.08 \pm 0.08 \pm 0.04 \pm 0.12) = 0.95(1 \pm 0.2) \qquad (3) \end{aligned}$$

The errors indicated in Eq. (3) are the measurement errors of the parameter A, ρ, m_t and $Br(K_L^0 \to \gamma\gamma)$ respectively and our theoretical uncertainty. On Fig.1 we represent the experimental data of the measurement of $Br(K_L^0 \to \mu^+\mu^-)$, where PDG, BNL and KEK are Particle Data Group, BNL E791 Collab. [13] and KEK E137 Collab. [14] results. To the left of the solid vertical line is the region which contradicts with the unitarity relation, to

the right of the dash vertical line is the region which agrees with the SM and B-physics. As we can see, the KEK result well correlates with the SM and B-physics, whereas the BNL results are in conflict with the SM. If the tendency of the quest for $Br(K_L^0 \to \mu^+\mu^-)$ to the unitarity limit will be verified by new series of more precise measurements it may be a signal of a new physics beyond the SM. For example, the real part of the total amplitude (2) can contain an extra term (the contribution of the relatively light leptoquark [15] or something else) which can cancel sufficiently the contribution of the top quark.

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